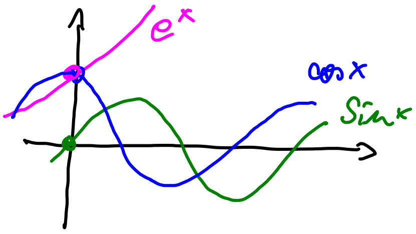
Vorlesung 14.10.21

Komplexe Zahlen II

Bew. $e^{i\varphi} = \cos\varphi + i\sin\varphi$



Taylorreihe

$$\sin\varphi = \varphi^1 - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} \dots$$

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} \dots$$

$$e^{\varphi} = 1 + \frac{\varphi}{1!} + \frac{\varphi^2}{2!} + \frac{\varphi^3}{3!} + \frac{\varphi^4}{4!} \dots$$

$$e^{i\varphi} = 1 + \frac{i\varphi}{1!} + \frac{(i\varphi)^2}{2!} + \frac{(i\varphi)^3}{3!} + \frac{(i\varphi)^4}{4!} \dots$$

$$= 1 + i\varphi - \frac{\varphi^2}{2!} - \frac{i\varphi^3}{3!} + \frac{\varphi^4}{4!} + \frac{i\varphi^5}{5!} \dots = \underline{\underline{\cos\varphi + i\sin\varphi}}$$

auch $e^{a+ib} = e^a \cdot e^{ib} = e^a \cdot (\cos b + i\sin b)$

analog $\boxed{\cos\varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}} = \frac{1}{2}\left(\cos\varphi + \underline{i\sin\varphi} + \cos(-\varphi) + \underline{i\sin(-\varphi)}\right)$

$\cos(-\varphi) = \cos\varphi$

$\sin(-\varphi) = -\sin\varphi$

auch für $\varphi \in \mathbb{C}$

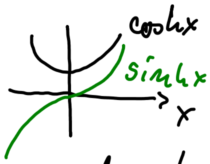
$$= \frac{1}{2}(2\cos\varphi) = \cos\varphi$$

und $\boxed{\sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}}$, Nachrechnen; Bem: $\frac{1}{i} = \quad | \cdot i$

$$= \frac{i}{i^2} = -i$$

auch $\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$



damit klar, warum diese Dinger mit h cos und sin bezeichnet werden

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$$e^{i(a+b)} = e^{ia} \cdot e^{ib} = (\cos a + i\sin a)(\cos b + i\sin b)$$

$$= \underline{\cos a \cdot \cos b - \sin a \sin b} + i\underline{(\cos a \sin b + \sin a \cos b)} \in \mathbb{C}$$

$$= \underline{\cos(a+b)} + i\underline{\sin(a+b)} =$$

$\Rightarrow$ $\left.\begin{array}{l} \cos(a+b) = \cos a \cos b - \sin a \sin b \\ \sin(a+b) = \cos a \sin b + \sin a \cos b \end{array}\right\}$ Additionstheoreme noch etwas Spaß $\quad \cos z = 2 \quad ; \quad z \in \mathbb{C}$

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} = 2$$

$$e^{iz}+e^{-iz} = 4 \qquad \Big|\; e^{iz} = t$$

$$t + \frac{1}{t} = 4 \qquad \Big|\cdot t$$

$$t^2 - 4t + 1 = 0$$

$$\Rightarrow t_{1/2} = 2 \pm \sqrt{3} = e^{iz} \qquad \Big|\ln$$

e^{iz} ist periodisch in 2π

$$iz = \ln(2 \pm \sqrt{3})$$

$$z = -i\ln(2 \pm \sqrt{3}) + 2\pi k \quad ; \quad k \in \mathbb{Z}$$

$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i\frac{\pi}{2}\cdot i} = e^{-\frac{\pi}{2}} = \frac{1}{\sqrt{e^{\pi}}} \in \mathbb{R}$$