Blatt 1

① a) $20x^2 \frac{3a}{5x} - \frac{a(x+6)}{3} = 12xa - \frac{ax+6a}{3}$

$= 12xa - \frac{ax}{3} - 2a$

$= \frac{35}{3}ax - 2a$

b) $\frac{a^2-b^2}{a+b} = \frac{(a-b)(a+b)}{a+b} = a-b$

c) $x^{3k+2} 3x^{4k+7} 7x^{n-9-7k} = 21x^{(3k+2+4k+7+n-9-7k)}$

$= 21x^4$

d) $\left(\frac{x^2y}{u^2v^2}\right)^4 : \left(\frac{xy^3}{u^2v}\right)^2 = \left(\frac{x^2y}{u^2v^2}\right)^4 \cdot \left(\frac{u^4 \cdot v^2}{x^2y^6}\right) = \frac{x^8y^4u^4v^2}{u^8v^8x^2y^6}$

$= \frac{x^6}{u^4v^6y^2}$

e) $(-a)^{-2} \cdot a = \frac{a}{(-a)^2} = \frac{1}{a}$

f) $-a^{-2} \cdot a = -\frac{a}{a^2} = -\frac{1}{a}$

g) $\sqrt[5]{32y^{10}} = 32^{1/5} \cdot y^2 = 2y^2$

h) $\sqrt[3]{\sqrt[4]{x^{24}}} = \left((x^{24})^{1/4}\right)^{1/3} = x^2$

i) $\frac{x-y}{y-x} = -\frac{(y-x)}{y-x} = -1$

j) $\frac{2-x}{4-x^2} + \frac{x+1}{x} - \frac{x+4}{x+2} - \frac{2}{x^2+2x}$

$4-x^2 = -(x-2)(x+2)$

$x^2+2x = x(x+2)$

$-[x^2+2x-2x-4]$

$$= \frac{2-x}{-(x-2)(x+2)} + \frac{x+1}{x} - \frac{x+4}{x+2} - \frac{2}{x(x+2)}$$

$$= \frac{1 \cdot x}{(x+2)\cdot x} + \frac{(x+1)(x+2)}{x\cdot(x+2)} - \frac{(x+4)x}{(x+2)x} - \frac{2}{(x+2)\cdot x}$$

$$= \frac{x + (x+1)(x+2) - x(x+4) - 2}{x(x+2)}$$

$$= \frac{x + x^2 + 3x + 2 - x^2 - 4x - 2}{x(x+2)} = 0$$

k) $$\frac{(x^2)^4 - x^{(2^4)}}{x^8} + x^8 = \frac{x^8 - x^{16}}{x^8} + x^8 = 1 - x^8 + x^8 = 1$$

l) $$\frac{x^{6n+2}\, x^{3-n}}{(x^2)^n\,(x^{n+3})^2} = \frac{x^{6n+2}\, x^{3-n}}{x^{2n}\, x^{2n+6}} = \frac{x^{6n+2+3-n}}{x^{2n+2n+6}} = \frac{x^{5n+5}}{x^{4n+6}}$$

$$x^{5n+5}\, x^{-(4n+6)} = x^{5n+5-4n-6} = x^{n-1}$$

(2)

a) $$\frac{2x-1}{2-x} = \frac{7}{3x+4} \qquad D = \mathbb{R} \setminus \{2, -4/3\}$$

$$(2x-1)(3x+4) = 7(2-x)$$
$$6x^2 + 8x - 3x - 4 = 14 - 7x$$
$$6x^2 + 12x - 18 = 0$$
$$x^2 + 2x - 3 = 0$$

p,q-Formel

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

$$x = -1 \pm \sqrt{1+3} = -1 \pm 2$$
$$x = \{-3, 1\}$$

b) $\frac{x+1}{2x-4} = \frac{x+2}{x-2}$ $\quad D = \mathbb{R} \setminus \{2\}$

$(x+1)(x-2) = (x+2)(2x-4) = (x+2)(x-2)\cdot 2$

$(x+1) = (x+2)2$

$x+1 = 2x+4$

$x = -3$

c) $2-3(7-4x) = 5x-7+2(4x+3)$

$2-21+12x = 5x-7+8x+6$

$-19 = x-1$

$x = -18$

d) $x(x-15)(x+23) = 0$ → Nullstellenform

$(x^2-15x)(x+23) = 0$

$x^3+23x^2-15x^2-345x = 0$

$x^3+8x^2-345x = 0$

$x_1 = 0 \quad \cup \quad x^2+8x-345 = 0$

$x = -4 \pm \sqrt{16+345} \Rightarrow x_2 = -23, x_3 = 15$

$x \in \{0, 15, -23\}$

e) $\frac{6x-1}{3x+2} = \frac{2x}{x-1}$ $\quad D = \mathbb{R} \setminus \{1, -2/3\}$

$(6x-1)(x-1) = 2x(3x+2)$

$6x^2-6x-x+1 = 6x^2+4x$

$11x = 1 \rightarrow x = 1/11$

f) $\log_{10}(3x+4) = 3$

$10^{\log_{10}(3x+4)} = 10^3$ $\rightarrow 3x+4 = 1000$

$\rightarrow x = 996/3 = 332$

g) $|x-1| \leq 1$

$$|x-1| = \begin{cases} x-1 & , x \geq 1 \\ 1-x & , x < 1 \end{cases}$$

① $x \geq 1$

$x-1 \leq 1 \rightarrow x \leq 2$

$x \geq 1 \wedge x \leq 2 \rightarrow x \in [1,2]$

② $x < 1$

$1-x \leq 1 \rightarrow x \geq 0$

$x < 1 \wedge x \geq 0 \rightarrow x \in [0,1)$

①② $\rightarrow x \in [0,1) \cup [1,2]$

$x \in [0,2]$

h) $\frac{4}{x-3} \leq 1$ $\quad D = \mathbb{R} \setminus \{3\}$

① $4 \leq x-3$ falls $x-3 > 0 \rightarrow x > 3$

$7 \leq x \rightarrow x \geq 7$

$x > 3 \wedge x \geq 7 \rightarrow x \in [7, \infty)$

② $4 \geq x-3$ falls $x-3 < 0 \rightarrow x < 3$

$7 \geq x$

$x \leq 7$

$x < 3 \wedge x \leq 7 \rightarrow x \in (-\infty, 3)$

$\rightarrow x \in (-\infty, 3) \cup [7, \infty) = \mathbb{R} \setminus [3,7)$

i) $\frac{x}{2x+1} < 2$ $\qquad D = \mathbb{R} \setminus \{-1/2\}$

① $2x+1 > 0 \rightarrow x > -1/2$

$\frac{x}{2x+1} < 2$

$x < 2(2x+1)$

$x < 4x+2$

$3x > -2$

$x > -2/3$

$x > -1/2 \wedge x > -2/3 \rightarrow x \in (-1/2, \infty)$

② $2x+1 < 0 \rightarrow x < -1/2$

$\frac{x}{2x+1} > 2$

$x > 2(2x+1) = 4x+2$

$3x < -2$

$x < -2/3$

$x < -1/2 \wedge x < -2/3 \rightarrow x \in (-\infty, -2/3)$

$\Rightarrow x \in (-\infty, -2/3) \cup (-1/2, \infty) = \mathbb{R} \setminus [-2/3, -1/2]$

j) $6 + \frac{1}{x+3} < 1$ $\qquad D = \mathbb{R} \setminus \{-3\}$

$\frac{1}{x+3} < -5$

① $x+3 > 0 \rightarrow x > -3$

$\frac{1}{x+3} < -5 \rightarrow 1 < -5(x+3)$

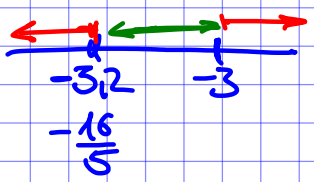
$1 < -5x - 15$

$16 < -5x$

$x < -16/5 \overset{=-3,2}{}$

$x > -3 \wedge x < -16/5$

$\rightarrow x \in \{\}$



② $x+3 < 0 \quad \rightarrow x < -3$

$\frac{1}{x+3} > -5 \quad \rightarrow 1 > -5x - 15$

$16/5 > -x$

$x > -16/5$

$x < -3 \wedge x > -16/5$

$\rightarrow x \in (-16/5; -3)$

k) $\frac{|1-x|}{x+3} \geq -2$

① $|1-x| \geq -2(x+3)$ falls $x+3 > 0 \rightarrow x > -3$

Ⓐ $|1-x| = 1-x$ falls $1-x \geq 0 \rightarrow x \leq 1$

$1-x \geq -2(x+3)$

$1-x \geq -2x-6$

$x \geq -7$

Ⓑ $|1-x| = x-1$ falls $x-1 \geq 0$ / $(1-x) < 0 \rightarrow x > 1$

$x-1 \geq -2(x+3)$

$x-1 \geq -2x-6$

$3x \geq -5$

$x \geq -5/3$

② $|1-x| \leq -2(x+3)$ falls $x+3 < 0 \rightarrow x < -3$

→ Ungleichheitszeichen drehen sich um

Ⓐ $x \leq 1 : x \leq -7$

Ⓑ $x > 1 : x \leq -5/3$

① Ⓐ $x > -3 \wedge x \leq 1 \wedge x \geq -7 \rightarrow x \in (-3, 1]$

Ⓑ $x > -3 \wedge x > 1 \wedge x \geq -5/3 \rightarrow x \in (1, \infty)$

② Ⓐ $x < -3 \wedge x \leq 1 \wedge x \leq -7 \rightarrow x \in (-\infty, -7]$

Ⓑ $x < -3 \wedge x > 1 \wedge x \leq -5/3 \rightarrow x \in \{\}$

$\Rightarrow x \in (-\infty, -7] \cup (-3, 1] \cup (1, \infty)$

$x \in (-\infty, -7] \cup (-3, \infty)$

$x \in \mathbb{R} \setminus (-7, -3]$

e) $\frac{2|x|}{x+3} \leq 1$ $\quad D = \mathbb{R} \setminus \{-3\}$

① $x+3 > 0 \rightarrow x > -3$

$2|x| \leq x+3$

Ⓐ $x \geq 0 \rightarrow |x| = x$

$2 \cdot x \leq x+3$

$x \leq 3$

Ⓑ $x < 0 \rightarrow |x| = -x$

$-2x \leq x+3$

$-3x \leq 3$

$x \geq -1$

② $x+3 < 0 \rightarrow x < -3$

$2|x| \geq x+3$

Ⓐ $x \geq 3$ for $x \geq 0$

Ⓑ $x \leq -1$ for $x < 0$

①Ⓐ $x > -3 \wedge x \geq 0 \wedge x \leq 3 \rightarrow x \in [0, 3]$

Ⓑ $x > -3 \wedge x < 0 \wedge x \geq -1 \rightarrow x \in [-1, 0)$

②Ⓐ $x < -3 \wedge x \geq 0 \wedge x \geq 3 \rightarrow x = \{\}$

Ⓑ $x < -3 \wedge x < 0 \wedge x \leq -1 \rightarrow x \in (-\infty, -3)$

$\rightarrow x \in (-\infty, -3) \cup [-1, 0) \cup [0, 3]$

$x \in (-\infty, -3) \cup [-1, 3]$

m) $|2x+4| \leq x+5$

① $2x+4 \geq 0 \qquad x \geq -2$

$2x+4 \leq x+5$

$x \leq 1$

$x \geq -2 \wedge x \leq 1 \rightarrow x \in [-2,1]$

② $2x+4 < 0 \qquad x < -2$

$-2x-4 \leq x+5$

$-3x \leq 9$

$x \geq -3$

$x < -2 \wedge x \geq -3 \rightarrow x \in [-3,-2)$

$\rightarrow x \in [-3,-2) \cup [-2,1]$

$x \in [-3,1]$

n) → Homepage

③ a) $V_1 = V_2$ $\qquad \rho = \frac{m}{V} \quad \left[\frac{kg}{dm^3}\right]$

$m \propto V$ $\qquad m \propto \rho$

↑ Proportional

$\frac{m_1}{m_2} = \frac{\rho_1 V_1}{\rho_2 V_2} \rightarrow \frac{m_1}{m_2} = \frac{\rho_1}{\rho_2}$

$m_2 = m_1 \frac{\rho_2}{\rho_1} = 4{,}8\,kg \cdot \frac{2{,}7\,kg/dm^3}{7{,}3\,kg/dm^3} \Rightarrow m_2 = 1{,}78\,kg$

b) $U = 2\pi R$

$V = \frac{4}{3}\pi R^3 \rightarrow V \propto U^3$

$m \propto V$

m_1 = Masse von N1 Kugeln mit der Dichte ρ_1

$$\frac{m_1}{m_2} = \frac{u_1^3}{u_2^3} \cdot \frac{N_1}{N_2} \Rightarrow$$

$$m_1 = 6{,}822\,kg$$

c) $2\,1/2\,a = 2{,}5$ Jahre

$$K_{2.5} = 10000€ \cdot 1{,}042 \cdot 1{,}042 \cdot 1{,}0.21$$
$$= 11085{,}65€$$

$$K_{20} = 10000€\,(1+z)^{20} \stackrel{!}{=} 2 \cdot 10000€$$
$$(1+z)^{20} = 2$$
$$1+z = 2^{1/20}$$

$$\Rightarrow 3{,}53\%$$

d) → entweder Homepage
oder: nächste Übung oder nächste Woche