

$$\textcircled{1} \text{ a) } \sqrt[24]{X^{33} / (X^3 (X^3)^2)} = \sqrt[24]{X^{27} / (X^9 X^6)} = (X^{27-9-6})^{1/24}$$

$$= X^{12/24} = X^{1/2} = \sqrt{X}$$

$$\text{b) } \frac{1}{X} + \frac{2x}{x-2} - \frac{5}{x-3} - \frac{20}{x^2+x-6} - \frac{3}{x^2+3x}$$

→ Linearfaktorzerlegung: $x^2+3x = x(x+3)$

$$x^2+x-6 = 0$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}} = -\frac{1}{2} \pm \frac{5}{2}$$

gemeinsamer Nenner: $x(x-2)(x+3)$

$$= \frac{(x-2)(x+3) + 2x \cdot x(x+3) - 5x(x-2) - 20 \cdot x - 3(x-2)}{x(x-2)(x+3)}$$

$$= \frac{x^2 - 2x + 3x - 6 + 2x^3 + 6x^2 - 5x^2 + 10x - 20x - 3x + 6}{x(x-2)(x+3)}$$

$$= \frac{2x^3 + 2x^2 - 12x}{x(x-2)(x+3)} = \frac{2x(x^2+x-6)}{x(x-2)(x+3)} = \frac{2x(x-2)(x+3)}{x(x-2)(x+3)} = 2$$

$$\text{c) } \sqrt{x+16} - \sqrt{x-12} = 2$$

$$D = \{x \in \mathbb{R} \mid x \geq -16 \wedge x \geq 12\}$$

$$= [12, \infty)$$

$$\sqrt{x+16} = 2 + \sqrt{x-12}$$

$$\Rightarrow x+16 = (2 + \sqrt{x-12})^2$$

nicht: $2^2 + (\sqrt{x-12})^2$ ✗

$$x+16 = 4 + 4\sqrt{x-12} + x-12$$

$$24 = 4\sqrt{x-12}$$

$$\sqrt{x-12} = 6$$

$$\Rightarrow x-12 = 36$$

$$x = 48$$

$$\rightarrow x \in D \rightarrow L = \{48\}$$

$$d) 8x^2 - 14x = 9$$

$$x^2 - \frac{14}{8}x - \frac{9}{8} = 0$$

$$x = \frac{7}{8} \pm \sqrt{\frac{49}{64} + \frac{72}{64}} = \frac{7}{8} \pm \frac{11}{8}$$

$$* x \in \left\{ \frac{9}{4}, -\frac{1}{2} \right\}$$

$$e) x^4 - \frac{7}{4}x^2 - \frac{9}{8} = 0$$

$$y = x^2 \geq 0$$

$$y^2 - \frac{7}{4}y - \frac{9}{8} = 0$$

$$\rightarrow y \in \left\{ \frac{9}{4}, -\frac{1}{2} \right\} \wedge y \geq 0$$

$$x^2 = \frac{9}{4} \rightarrow x = \pm \frac{3}{2}$$

$$x \in \left\{ -\frac{3}{2}, \frac{3}{2} \right\}$$

$$f) |x+1| + |x+2| \leq 2$$

$$\textcircled{1} x+1 \geq 0 \rightarrow \underline{x \geq -1}$$

$$\textcircled{A} x+2 \geq 0 \rightarrow x \geq -2$$

$$(x+1) + (x+2) \leq 2$$

$$2x \leq -1$$

$$x \leq -\frac{1}{2}$$

$$\rightarrow x \geq -1 \wedge x \geq -2 \wedge x \leq -\frac{1}{2}$$

$$\rightarrow x \in [-1, -\frac{1}{2}]$$

$$\textcircled{B} x+2 < 0 \rightarrow x < -2$$

$$\rightarrow x \geq -1 \wedge x < -2$$

$$\rightarrow x \in \{ \}$$

$$\textcircled{2} x+1 < 0 \rightarrow x < -1$$

$$\textcircled{A} x+2 \geq 0 \rightarrow x \geq -2$$

$$(-x-1) + (x+2) \leq 2 \rightarrow -1 \leq 2$$

$$\rightarrow x < -1 \wedge x \geq -2$$

$$\rightarrow x \in [-2, -1)$$

$$\textcircled{3} x+2 < 0 \rightarrow x < -2$$

$$(-x-1) + (-x-2) \leq 2$$

$$-2x \leq 5$$

$$x \geq -5/2$$

$$\rightarrow x < -1 \wedge x < -2 \wedge x \geq -5/2$$

$$\rightarrow x \in [-5/2, -2)$$

$$\Rightarrow x \in [-1, -1/2] \cup [-2, -1) \cup [-5/2, -2)$$

$$x \in [-5/2, -1/2]$$

$$g) \frac{x+2}{x^2-x-2} < -1$$

$$\textcircled{1} x^2-x-2 > 0$$

$$x^2-x-2=0$$

$$x = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}} = \frac{1}{2} \pm \frac{3}{2}$$

$$x = \{-1, 2\}$$

Parabel nach oben geöffnet

$$x^2-x-2 > 0 \text{ falls } x \in \mathbb{R} \setminus [-1, 2]$$

$$x+2 < -x^2+x+2$$

$$x^2 < 0$$

$$x \in \{\}$$

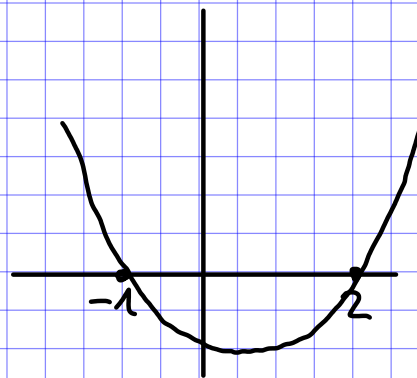
$$\textcircled{2} x^2-x-2 < 0$$

$$x \in (-1, 2)$$

$$x^2 > 0$$

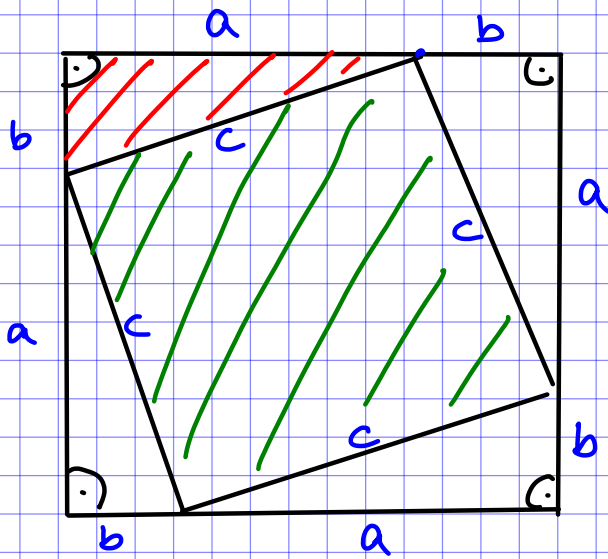
$$x \in \mathbb{R} \setminus \{0\}$$

$$x \in (-1, 2) \cup \mathbb{R} \setminus \{0\} \quad x \in (-1, 0) \cup (0, 2)$$



②

a)

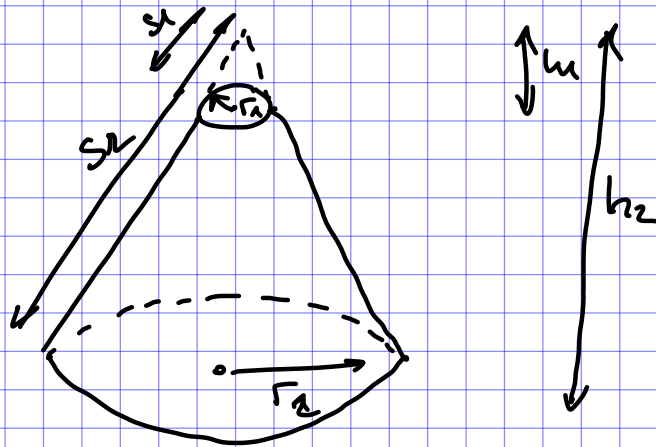
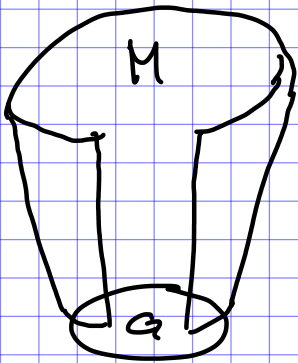


$$(a+b)^2 = 4 \cdot \frac{a \cdot b}{2} + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

b)



$$V = \frac{h\pi}{3} (r_1^2 + r_1 r_2 + r_2^2) \approx 10,5L$$

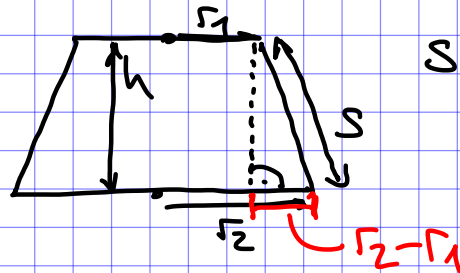
$$\text{mit } 1 \text{ cm}^3 = (10^{-2})^3 \text{ m}^3 = 10^{-6} \text{ m}^3$$

$$1 \text{ m}^3 = 1000L$$

$$1 \text{ cm}^3 = 10^{-6} \text{ m}^3 \cdot \frac{10^3 L}{\text{m}^3} = 10^{-3} L$$

Mantelfläche

$$M = (r_2 + r_1) \pi \cdot s$$



$$s = \sqrt{(r_2 - r_1)^2 + h^2}$$

$$M = 0,199672 \text{ m}^2$$

Materialverbrauch

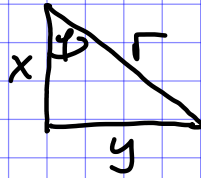
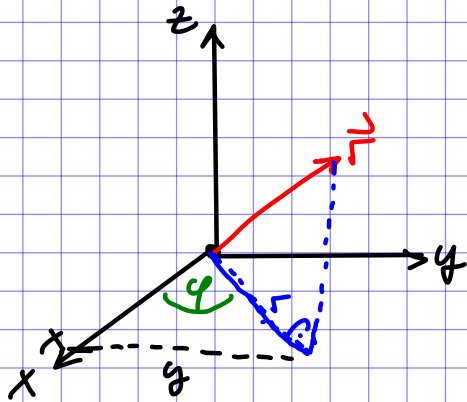
$$M + \pi r_1^2 = 19,9672 \text{ dm}^2 + 2,2698 \text{ dm}^2$$

$$= 22,237 \text{ dm}^2$$

c)

$$\vec{r} = \begin{pmatrix} r \cos \varphi \\ r \sin \varphi \\ z \end{pmatrix}$$

$$\begin{aligned} x &= r \cos \varphi \\ y &= r \sin \varphi \\ z &= z \end{aligned}$$



$$r^2 = x^2 + y^2$$

$$\tan \varphi = y/x$$

$$(r, \varphi, z)$$

$$\frac{y}{x} = \frac{r \sin \varphi}{r \cos \varphi} = \frac{\sin \varphi}{\cos \varphi}$$

a) $(1, 0, 0)$ $r=1$, $\varphi=0$, $z=0$
 $\rightarrow (1, 0, 0)$

b) $(1, 1, 0)$ $r=\sqrt{2}$, $\varphi=45^\circ$, $z=0$
 $\rightarrow (\sqrt{2}, 45^\circ, 0)$

c) $(1, 1, 1)$ $r=\sqrt{2}$ $\varphi=45^\circ$, $z=1$
 $\rightarrow (\sqrt{2}, 45^\circ, 1)$

d) $(1, \sqrt{2}, 0)$ $z=0$
 $\rightarrow (\sqrt{3}, 54,74^\circ, 0)$

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$$

$$\varphi = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\sqrt{2}}{1}\right) = 54,74^\circ$$

e) $(2, 3, 4)$ $z=4$
 $\rightarrow (\sqrt{13}, 56,31^\circ, 4)$

$$r = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\varphi = \arctan\left(\frac{3}{2}\right)$$

f) $(0, 0, 0) \rightarrow (0, 0, 0)$

③ a) 2. Newton Axiom

$$F = m \cdot a$$

$$[F] = [m][a] = 1 \text{ kg} \frac{\text{m}}{\text{s}^2} = 1 \text{ N}$$

Einheit der Kraft

b) $E_{\text{kin}} = \frac{m v^2}{2}$

$$[E] = 1 \text{ kg} \frac{\text{m}^2}{\text{s}^2} = 1 \text{ J}$$

Einheit der Energie

c) $\vec{E} = \frac{q}{4\pi \epsilon_0} \frac{\vec{r}}{r^3}$

\uparrow
 $\frac{\text{As}}{\text{Vm}}$

$$[E] = \frac{1 \text{ C}}{\frac{\text{As}}{\text{Vm}}} \frac{\text{m}}{\text{m}^3} = \frac{1 \text{ C m Vm}}{\text{C/s} \cdot \text{s m}^3} = \frac{\text{V}}{\text{m}}$$

d) \rightarrow in dm^2

a) $235 \text{ km}^2 = 235 \cdot (10^3)^2 \text{ dm}^2$ $1 \text{ km} = 10^3 \text{ m} = 10 \cdot 10^3 \text{ dm} = 10^4 \text{ dm}$

$$= 235 \cdot 10^6 \text{ dm}^2 = 2,35 \cdot 10^{10} \text{ dm}^2$$

b) $0,287 \text{ m}^2 = 0,287 (10^1)^2 \text{ dm}^2 = 28,7 \text{ dm}^2$

c) $342748 \text{ mm}^2 = 3,42748 \cdot 10^5 \cdot (10^{-2})^2 \text{ dm}^2$

$$= 34,2748 \text{ dm}^2$$

$$1 \text{ mm} = 10^{-2} \text{ dm}$$

d) $8,342 \cdot 10^4 \text{ mm} \cdot \text{m}$ $= 8,342 \cdot 10^3 \text{ dm}^2$

$\underbrace{10^{-2} \cdot 10^1 \text{ dm}^2}_{10^{-1} \text{ dm}^2}$

e) $3,648 \cdot 10^{17} \text{ } \mu\text{m}^2 = 3,648 \cdot 10^7 \text{ dm}^2$

\uparrow
 $(10^{-6})^2 \text{ m}^2$

$$(10^{-6})^2 \cdot 10^2 \text{ dm}^2 = 10^{-10} \text{ dm}^2$$

$$f) 2 \mu\text{m} \cdot \frac{\text{m}}{\text{km}} \cdot \frac{\text{dm}}{\text{km}} = 2 \cdot 10^{-5} \cdot 10 \cdot \frac{1}{10^4} \text{ dm}^2$$

$$= 2 \cdot 10^{-8} \text{ dm}^2$$

$10^{-6} \text{ m} \cdot 10 \text{ dm} = 10^{-5} \text{ dm}$
 $\frac{1 \text{ dm}}{10^3 \text{ m}} = 10^{-3} \text{ m}^{-1}$
 $\frac{1 \text{ dm}}{10^4 \text{ dm}} = 10^{-4} \text{ dm}^{-1}$

→ in cm^3

a) $9,837 \cdot 10^{19} \text{ nm}^3$
 $= 9,837 \cdot 10^{-2} \text{ cm}^3$

$1 \text{ nm} = 10^{-9} \text{ m}$
 $1 \text{ nm}^3 = (10^{-9})^3 \text{ m}^3$
 $= (10^{-9})^3 (10^2)^3 \text{ cm}^3$
 $= 10^{-27} \cdot 10^6 \text{ cm}^3 = 10^{-21} \text{ cm}^3$

b) $5,32 \cdot 10^4 \text{ mL}$
 $= 5,32 \cdot 10^4 \text{ cm}^3$

$1 \text{ L} = 10^{-3} \text{ m}^3$
 $= 10^{-3} \cdot (10^2)^3 \text{ cm}^3$
 $= 10^3 \text{ cm}^3$

c) $0,0345 \text{ m}^3 = 0,0345 \cdot 10^6 \text{ cm}^3$
 $= 3,45 \cdot 10^4 \text{ cm}^3$

d) $2 \mu\text{m} \text{ dm} \text{ km} = 2 \cdot 10^{-4} \cdot 10 \cdot 10^5 \text{ cm}^3 = 2 \cdot 10^2 \text{ cm}^3$

$10^{-6} \text{ m} \cdot 10^2 \text{ cm} = 10^{-4} \text{ cm}$
 $10^3 \text{ m} = 10^5 \text{ cm}$

e) $4370 \text{ mm} \frac{\text{L}}{\text{cm}} = 4370 \cdot 10^{-1} \cdot 10^3 \text{ cm}^3$
 $= 4,370 \cdot 10^3 \cdot 10^{-1} \cdot 10^3 \text{ cm}^3$
 $= 4,370 \cdot 10^5 \text{ cm}^3$

$1 \text{ L} = 10^{-3} \text{ m}^3$
 $= 10^{-3} (10^2)^3 \text{ cm}^3$
 $= 10^3 \text{ cm}^3$

f) $0,45 \frac{\mu\text{m}^2}{\text{dm}} \cdot \frac{\text{km}^2}{\text{mm}} \cdot \text{cm} = 0,45 \cdot \frac{10^{-8}}{10} \cdot \frac{10^{10}}{10^{-7}} \text{ cm}^3 = 0,45 \cdot 10^8 \text{ cm}^3$
 $4,5 \cdot 10^7 \text{ cm}^3$

$(10^{-6})^2 \text{ m}^2 = 10^{-12} (10^4)^2 \text{ cm}^2 = 10^{-8} \text{ cm}^2$
 $1 \text{ km}^2 = (10^3)^2 \text{ m}^2 = 10^6 (10^2)^2 \text{ cm}^2 = 10^{10} \text{ cm}^2$
 $1 \text{ mm} = 10^{-3} \text{ m} = 10^{-9} \cdot 10^2 \text{ cm} = 10^{-7} \text{ cm}$