azixmey

1) a) $20x^2 \frac{3\alpha}{5x} - \frac{\alpha(x+6)}{3}$

$= \frac{20 \cdot 3}{5} \frac{x^2 \alpha}{x} - \frac{\alpha x + 6\alpha}{3}$

$= 12 x \alpha - \frac{\alpha x}{3} - 2\alpha$

$= 12 x \alpha \cdot \frac{3}{3} - \frac{1\alpha x}{3} - 2\alpha$

$= \frac{35}{3} x \alpha - 2\alpha = \alpha\left(\frac{35}{3} x - 2\right)$

b) $\frac{\alpha^2 - \beta^2}{\alpha + \beta}$ ← 3. bin. Formel

$= \frac{(\alpha - \beta) \cdot (\alpha + \beta)}{(\alpha + \beta)} = \alpha - \beta$

c) $x^{3k+2} \cdot 3x^{4k+7} \cdot 7x^{n-9-7k}$ $\quad k = \gamma$

$= 3 \cdot 7 \; x^{3k+2+4k+7+n-9-7k}$

$= 21 \cdot x^n$

d) $\left(\frac{x^2 y}{u^2 v^2}\right)^4 : \left(\frac{xy^3}{u^2 v}\right)^2$

$= \left(\frac{x^2 y}{u^2 v^2}\right)^4 \cdot \left(\frac{u^2 v}{xy^3}\right)^2$

$= \frac{x^8 y^4}{u^8 v^8} \cdot \frac{u^4 v^2}{x^2 y^6}$

$$= \frac{x^8}{x^2} \cdot \frac{y^4}{y^6} \cdot \frac{u^4}{u^8} \cdot \frac{v^2}{v^8}$$

$$= \underbrace{x^{8-2}}_{= x^8 \cdot x^{-2}} \cdot y^{-2} \cdot u^{-4} \cdot v^{-6}$$

$$= \frac{x^6}{y^2 u^4 v^6}$$

e) $(-a)^{-2} a = \frac{1}{(-a)^2} a = \frac{1}{a^2} \cdot a = \frac{1}{a}$

$(-a) \cdot (-a) = a^2$

f) $-a^{-2} a = -\frac{1}{a^2} \cdot a = -\frac{1}{a}$

g) $\sqrt[5]{32 \Sigma^{10}} = (32 \Sigma^{10})^{\frac{1}{5}} = 32^{\frac{1}{5}} \cdot \Sigma^{\frac{10}{5}}$

$= 32^{\frac{1}{5}} \cdot \Sigma^2$

$= 2 \Sigma^2$

h) $\sqrt[3]{\sqrt[4]{\Gamma^{24}}} = \left((\Gamma^{24})^{\frac{1}{4}} \right)^{\frac{1}{3}} = \Gamma^{24 \cdot \frac{1}{12}} = \Gamma^2$

i) $\frac{\overbrace{\mu - \nu}^{+ \; -}}{\underbrace{\nu - \mu}_{+ \; -}} \cdot \frac{(-1)}{(-1)} = \frac{-1(\mu - \nu)}{-1(\nu - \mu)}$

$= \frac{(\nu - \mu)}{-1(\nu - \mu)} = \frac{1}{(-1) \cdot 1} = -1$

j) $\frac{2 - x}{\underbrace{4 - x^2}_{2^2 - x^2 = (2-x)(2+x)}} + \frac{x+1}{x} - \frac{x+4}{x+2} - \frac{2}{x^2 + 2x}$

$$= \frac{1}{2+x} + \frac{x+1}{x} - \frac{x+4}{x+2} - \frac{2}{x^2+2x}$$

$$= \frac{\overbrace{1-x-4}^{-3-x}}{x+2} + \frac{x+1}{x} - \frac{2}{x^2+2x}$$

$$= \frac{x}{x} \cdot \frac{-3-x}{x+2} + \frac{x+1}{x} - \frac{2}{x^2+2x}$$

$$= \frac{-3x-x^2}{x^2+2x} - \frac{2}{x^2+2x} + \underbrace{\frac{x}{x}}_{=1} + \frac{1}{x}$$

$$= \frac{-2-3x-x^2}{\underbrace{x^2+2x}_{x(x+2)}} + \frac{x+1}{x}$$

$$= \frac{-2-3x-x^2}{x(x+2)} \quad \frac{(x+1)}{x} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{-2-3x-x^2+x^2+3x+2}{x(x+2)} = 0$$

k) $$\frac{(\varepsilon^2)^4 - \varepsilon^{(2^4)}}{\varepsilon^8} + \varepsilon^8$$

$$= \frac{\varepsilon^{2\cdot 4} - \varepsilon^{16}}{\varepsilon^8} + \varepsilon^8$$

$$= \frac{\varepsilon^8}{\varepsilon^8} - \frac{\varepsilon^{16}}{\varepsilon^8} + \varepsilon^8$$

$= 1 - \varepsilon^8 + \varepsilon^8 = 1$

l) $\dfrac{\lambda^{6n+2}\,\lambda^{3-n}}{(\lambda^2)^n(\lambda^{n+3})^2}$

$= \dfrac{\lambda^{6n+2+3-n}}{\lambda^{2n+(n+3)\cdot 2}} = \dfrac{\lambda^{5n+5}}{\lambda^{4n+6}}$

$= \lambda^{5n+5} \cdot \lambda^{-4n-6} = \lambda^{5n-4n+5-6}$

$= \lambda^{n-1}$

A2) a) $\dfrac{2x-1}{2x} = \dfrac{7}{3x+4}$ $\quad D = \mathbb{R}\setminus\{2, -\frac{4}{3}\}$

$\Leftrightarrow (2x-1)(3x+4) = 7(2-x)$

$\Leftrightarrow 6x^2 + 8x - 3x - 4 = 14 - 7x \quad | + 7x \; | - 14$

$\Leftrightarrow 6x^2 + \underbrace{15x - 3x}_{=12x} - 18 = 0 \quad | : 6$

$\Leftrightarrow x^2 + 2x - 3 = 0 \quad | + 4$

$\Leftrightarrow x^2 + 2x + 1 = 4$

$\Leftrightarrow (x+1)^2 = 4 \quad | \sqrt{\;}$

$\Leftrightarrow x + 1 = 2 \quad \vee \quad x + 1 = -2$

$\Leftrightarrow x = 1 \quad \vee \quad x = -3$

b) $\frac{x+1}{2x-4} = \frac{x+2}{x-2}$ $\quad D = \mathbb{R} \setminus \{2\}$

$\Leftrightarrow (x+1)\cdot(x-2) = (x+2)\cdot(2x-4)$

$\Leftrightarrow (x+1)\cdot(x-2) = (x+2)(x-2)\cdot 2 \quad | : (x-2)$

$\Leftrightarrow (x+1) = (x+2)\cdot 2$

$\Leftrightarrow x = -3$

c) $2-3(7-4x) = 5x-7+2(4x+3) \quad D = \mathbb{R}$

$2-21+12x = 5x-7+8x+6$

$-18 = x$

d) $x(x-15)(x+23) = 0$

$x=15 \quad x=-23$

$(x^2-15x)(x+23) = 0$

$x = 15 \vee x = 0 \vee x \quad x = -23$

e) $\frac{6x-1}{3x+2} = \frac{2x}{x-1} \quad D = \mathbb{R} \setminus \{-\frac{2}{3}, 1\}$

$(6x-1)\cdot(x-1) = 2x\cdot(3x+2)$

$6x^2-6x-x+1 = 6x^2+4x \quad | -6x^2$

$-6x-x+1 = 4x$

$-7x+1 = 4x$

$11x = 1$

$x = \frac{1}{11}$

f) $\log_{10}(3x+4) = 3$ $\qquad$ $\log_2(8) = 3$

$3x+4 = 10^3$ $\qquad$ $2^x = 8 \mid \log_2$

$3x+4 = 1000 \quad | -4 | :3$ $\qquad$ $\log_2(8) = x = 3$

$x = 332$

g) $|x-1| \leq 1$

$x \geq 1$:

$x - 1 \leq 1$

$x = 2$ obere Grenze

$x = 1$ untere Grenze

$\Rightarrow x \in [1, 2]$

$x = 0$:

$|0 - 1| \leq 1$

$|-1| = 1$

$\Rightarrow x \in [0, 2]$

$x < 0$:

$|\underbrace{x-1}_{<-1}| \leq 1$ für alle $x < 0$

$|x-1| \not\leq 1$

$\Rightarrow x \in [0, 2]$

$|\cdot| = \sqrt{(\cdot)^2}$

$\sqrt{(x-1)^2} \leq 1$

$(x-1)^2 \leq 1$

$-1 \leq (x-1) \leq 1 \quad | +1$

$0 \leq x \leq 2$

$\Rightarrow x \in [0, 2]$

h) $\frac{4}{x-3} \leq 1$ $\quad D = \mathbb{R} \setminus \{3\}$

① $4 \leq 1 \cdot (x-3)$

$4 \leq x-3$ $\quad x-3 > 0 \Rightarrow x > 3$

$\Rightarrow x \geq 7$ $\quad x \geq 7 \wedge x > 3$

$\Rightarrow x \geq 7$

$\Rightarrow x \in [7, \infty)$

② $4 \geq x-3$ $\quad x-3 < 0 \Rightarrow x < 3$

$x \leq 7$ $\quad x \leq 7 \wedge x < 3$

$\Rightarrow x \in (-\infty, 3)$

①∨② $x \in (-\infty, 3) \cup [7, \infty) = \mathbb{R} \setminus [3, 7)$

i) $\frac{x}{2x+1} < 2$ $\quad D = \mathbb{R} \setminus \{-\frac{1}{2}\}$

① $2x+1 > 0 \Rightarrow x > -\frac{1}{2}$

$x < 4x+2 \Rightarrow 3x > -2 \Rightarrow x > -\frac{2}{3}$

$\Rightarrow x > -\frac{1}{2}$

$\Rightarrow x \in (-\frac{1}{2}, \infty)$

② $2x+1 < 0 \Rightarrow x < -\frac{1}{2}$

$x > 4x+2 \Rightarrow x < -\frac{2}{3}$

$\Rightarrow x \in (-\infty, -\frac{2}{3})$

② + ① $\Rightarrow$ $x \in (-\infty, -\frac{2}{3}) \cup (-\frac{1}{2}, \infty) = \mathbb{R} \setminus [-\frac{2}{3}, -\frac{1}{2}]$

j) $6 + \frac{1}{x+3} < 1$ $\quad$ I-6 $\quad D = \mathbb{R} \setminus \{-3\}$

$\frac{1}{x+3} < -5$

① : $x+3 > 0 \Rightarrow x > -3$

$1 < -5x - 15 \Rightarrow 5x < -16 \Rightarrow x < -\frac{16}{5}$

$x \in \{\}$

② : $x + 3 < 0 \Rightarrow x < -3$

$x > -\frac{16}{5}$ $\quad -\frac{16}{5} < -3$

$\Rightarrow x \in (-\frac{16}{5}, -3)$

① + ② $\Rightarrow x \in (-\frac{16}{5}, -3)$

k) $\frac{|1-x|}{x+3} \geq -2$

① $|1-x| \geq -2(x+3)$ $\quad , x+3 > 0 \Rightarrow x > -3$

Ⓐ $|1-x| = 1-x$, $1-x \geq 0$, $x \leq 1$

$1-x \geq -2(x+3)$

$1-x \geq -2x-6 \Rightarrow x \geq -7$

Ⓑ $|1-x| = x-1$ $\quad x-1 > 0 \Rightarrow x > 1$

$x-1 \geq -2(x+3) \Rightarrow x-1 \geq -2x-6$

$\Rightarrow 3x \geq -5 \Rightarrow x \geq -\frac{5}{3}$

② $(1-x| \leq -2(x+3)$ $\quad x+3 < 0 \Rightarrow x < -3$

Ⓐ $x \leq 1$: $x \leq -7$

Ⓑ $x > 1$: $x \leq -\frac{5}{3}$

① ∪ ② $\Rightarrow x \in (-\infty, -7] \cup (-3, 1] \cup (1, \infty)$

$= (-\infty, -7] \cup (-3, \infty) = \mathbb{R} \setminus (-7, -3]$

l) $\frac{2|x|}{x+3} \leq 1$ $D = \mathbb{R} \setminus \{-3\}$

① $x+3 > 0 \Rightarrow x > -3$

$2|x| \leq x+3$

Ⓐ $x \geq 0 \Rightarrow |x| = \sqrt{x^2} = x$

$2x \leq x+3 \Rightarrow x \leq 3$

$-x$

Ⓑ $x < 0 \Rightarrow (|x| = -x)$

$-2x \leq x+3 \Rightarrow x \geq -1$

② $x+3 < 0 \Rightarrow x < -3$

Ⓐ $|x| = x$, $x \geq 0$

$x \geq 3$

Ⓑ $|x| = -x$, $x < 0$

$x \leq -1$

① (A) $x > -3 \wedge x \leq 3 \wedge x \geq 0 \Rightarrow x \in [0,3]$

(B) $x > -3 \wedge x < 0 \wedge x \geq -1 \Rightarrow x \in [-1,0)$

② (A) $x < -3 \wedge x \geq 0 \wedge x \geq 3 \Rightarrow x \in \{\}$

(B) $x < -3 \wedge x < 0 \wedge x \leq -1 \Rightarrow \in (-\infty, -3)$

$\Rightarrow \quad x \in (-\infty, -3) \cup [-1,0) \cup [0,3]$

$= (-\infty, -3) \cup [-1,3]$

m) $x \in [-3,-2) \cup [-2,1] = [-3,1]$

n) $x \in (-\infty, -5] \cup [-\frac{3}{2}, -3)$

3) a) $A = 7,3 \frac{kg}{dm^3}$ $\quad B = 2,7 \frac{kg}{dm^3}$

$M_A = 4,8\,kg$ $\quad M_B = ?$

$V_A = V_B$

$$\frac{2,7 \frac{kg}{dm^3}}{7,3 \frac{kg}{dm^3}} = \frac{M_B}{M_A}$$

$$\frac{2,7}{7,3} \cdot M_A \approx 1,78\,kg$$

b) 15 Kugeln, $u = 70\,cm = 0,7\,m$

$M_k = 6,5\,kg$

25 Kugeln $\quad u' = 60\,cm = 0{,}6\,m$

$u = 2\pi R$

$V = \frac{4}{3}\pi R^3$ $\Rightarrow V \propto u^3$

$M_K' \propto V N$ $\Rightarrow$ $\frac{M_K}{M_K'} = \frac{u^3}{u'^3} \cdot \frac{N}{N'}$

$M_K' \propto u^3 N$

$N = 15$ Kugeln

$N' = 25$ Kugeln

$M_K' = 6{,}822\,kg$

c) $K = 10000\,€$ für 2,5 a bei 4,2% Verzinst.

2 a

$K_{2,5} = K_2 = 10000\,€ \cdot 1{,}042 \cdot 1{,}042$

$= 10857{,}64\,€$

Verdopplung nach 20 a, welcher Zinssatz?

$K_{20} = 10000\,€ \cdot (1+z)^{20} \overset{!}{=} 20000\,€$

$10000 \cdot (1+z)^{20} = 20000 \quad | : 10000$

$(1+z)^{20} = 2 \quad | \sqrt[20]{\ }$

$1+z = 2^{\frac{1}{20}}$

$z = 2^{\frac{1}{20}} - 1 = 0{,}0353$

$z \approx 3{,}53\%$

d) 100 : 2 Möglichkeiten:

$100 = 25 \cdot 4 = 5 \cdot 5 \cdot 2 \cdot 2$

i) $100 = \underbrace{4}_{5 \cdot 5 = 25} \cdot \underbrace{2}_{}$ Mitte der Zahlenfolge $z \in \mathbb{Z}$

$5 \cdot 5 \cdot 2 = 50$

$2 \cdot 2 = 4$

$2 \cdot 2 \cdot 5 = 20$