1) a) $\sqrt[24]{x^3 / (x^3)^2 (x^3)^2} = (x^3 \cdot x^{-3^2} \cdot (x^3)^{-2})^{\frac{1}{24}}$

$= (x^{3^3 - 3^2 + 6})^{\frac{1}{24}}$

$= (x^{27 - 9 + 6})^{\frac{1}{24}}$

$= x^{\frac{1}{2}} = \sqrt{x}$

b) $\frac{1}{x} + \frac{2x}{x-2} - \frac{5}{x+3} - \frac{20}{x^2+x-6} - \frac{3}{x^2+3x}$

$x^2 + x - 6 = (x-2)(x+3)$

$x^2 + 3x = x(x+3)$

$\frac{x}{x}$ $\quad$ $\frac{(x-2)}{(x-2)}$

$\frac{1}{x} + \frac{2x}{x-2} - \frac{5}{x+3} - \frac{20x - 3x + 6}{x(x-2)(x-3)}$

$\frac{(x-2)(x+3)}{(x-2)(x+3)}$ $\quad$ $\frac{x(x+3)}{x(x+3)}$ $\quad$ $\frac{x \cdot (x-2)}{x(x-2)}$

$$\frac{(x-2)(x+3) + 2x \cdot x(x+3) - 5x \cdot (x-2) - 20x - 3x + 6}{x \cdot (x-2)(x-3)}$$

$$= \frac{x^2 - 2x + 3x - 6 + 2x^3 \ \ldots\ldots}{x(x-2)(x-3)} = 2$$

$$= \frac{2x^3 + 2x^2 - 12x}{x(x-2)(x-3)} = \frac{2x(x^2+x-6)}{x(x^2+x-6)} = 2$$

c) $$\frac{\frac{a}{a-b} - \frac{b}{a+b}}{\frac{a}{a+b} + \frac{b}{a-b}} = \frac{\frac{a(a+b) - b(a-b)}{(a-b)(a+b)}}{\frac{a(a-b) + b(a+b)}{(a+b)\cdot(a-b)}}$$

$$= \frac{a(a+b) - b(a-b)}{(a-b)(a+b)} \cdot \frac{(a+b)(a-b)}{a(a-b) + b(a+b)}$$

$$= \frac{a(a+b) - b(a-b)}{a(a-b) + b(a+b)}$$

$$= \frac{a^2 + ab - ba + b^2}{a^2 - ab + ba + b^2}$$

$$= \frac{a^2 + b^2}{a^2 + b^2} = 1$$

d) $\ln \sqrt{e^{3(\ln e^2 + \ln e^6)}}$

$\ln = \log_e$

$\ln e^2 = \log_e e^2 = 2$

$\ln e^6 = 6$

$\Big\} 2+6 = 8$

$\ln \sqrt{e^{3 \cdot 8}} = \ln e^{12}$

$= \log_e e^{12}$

$= 12$

e) $\sqrt{x+16} - \sqrt{x-12} = 2$ $\quad D = \{x \in \mathbb{R} : x \geq -16 \wedge x \geq 12\}$

$= [12, \infty)$

$\sqrt{x+16} = 2 + \sqrt{x-12} \quad | ()^2$

$x + 16 = (2 + \sqrt{x-12})^2$

$x + 16 = 4 + 4\sqrt{x-12} + x - 12$

$24 = 4\sqrt{x-12}$

$\sqrt{x-12} = 6 \quad \Rightarrow \quad x - 12 = 36 \quad | -36$

$\Rightarrow x = 48 \in D$ (nicht möglich wegen D)

$\Rightarrow \mathbb{L} = \{48\}$

f) $x \in \{\frac{9}{4}, -\frac{1}{2}\}$

g) $x^4 - \frac{7}{4}x^2 - \frac{9}{8} = 0$

$y = x^2 \geq 0$ ←

$y^2 - \frac{7}{4}y - \frac{9}{8} = 0$

$y \in \{\frac{9}{4}, -\frac{1}{2}\}$ $\wedge$ $y \geq 0$

$x^2 = \frac{9}{4} \Rightarrow x = \pm\frac{3}{2} \Rightarrow x \in \{\frac{3}{2}, -\frac{3}{2}\}$

h) $|x+1| + |x+2| \leq 2$

① $x+1 \geq 0 \rightarrow x \geq -1$ /

Ⓐ $x+2 \geq 0 \rightarrow x \geq -2$ /

$x+1+x+2 \leq 2$

$2x+3 \leq 2$

$2x \leq -1 \Rightarrow x \leq -\frac{1}{2}$ /

$x \geq -1 \wedge x \geq -2 \wedge x \leq -\frac{1}{2}$

$\rightarrow x \in [-1, -\frac{1}{2}]$

Ⓑ $x+2 < 0 \rightarrow x < -2$

$x \geq -1 \wedge x < -2 \Rightarrow x \in \{\}$

② $x+1 < 0 \rightarrow x < -1$

Ⓐ $x+2 \geq 0 \rightarrow x \geq -2$

$(-x-1) + x + 2 \leq 2 \rightarrow +1 \leq 2$

$x < -1 \quad \wedge \quad x \geq -2$

$x \in [-2, -1)$

(B) $x + 2 < 0 \rightarrow x < -2$

$-x - 1 - x - 2 \leq 2$

$-2x \leq 5$ $\qquad 1 - \frac{1}{2}$

$x \geq -\frac{5}{2}$

$x \in [-\frac{5}{2}, -2)$

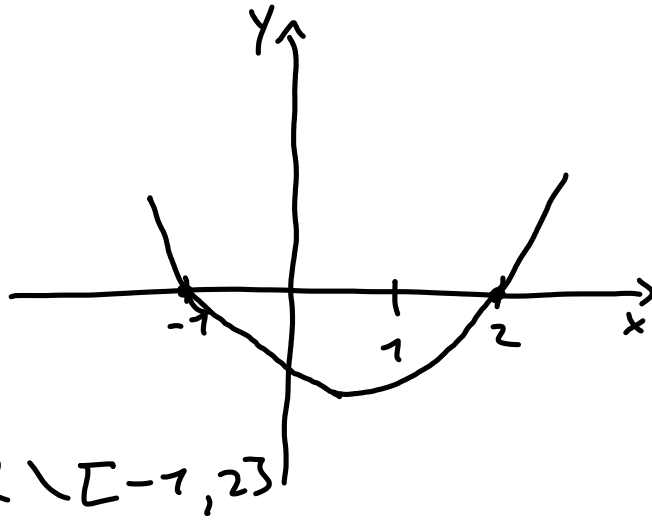
$\Rightarrow x \in [-1, -\frac{1}{2}] \cup [-2, -1) \cup [-\frac{5}{2}, -2)$

$x \in [-\frac{5}{2}, -\frac{1}{2}]$

i) $\dfrac{x+2}{x^2 - x - 2} < -1$

$x^2 - x - 2 = 0$

$x = \{-1, 2\}$



(1) $x^2 - x - 2 > 0 \qquad x \in \mathbb{R} \setminus [-1, 2]$

$x + 2 < -x^2 + x + 2$

$x^2 < 0 \quad -$

$x \in \{\}$

(2) $x^2 - x - 2 < 0 \qquad x \in (-1, 2)$

$x^2 > 0$

$x \in \mathbb{R} \setminus \{0\}$

$x \in \mathbb{R} \setminus \{0\} \cup \quad x \in (-1, 2)$

$\Rightarrow x \in (-1, 0) \cup (0, 2)$

~~X~~2 $l = a + b \qquad a > b$

$\frac{l}{a} = \frac{a}{b}$

$\frac{l}{a} = \frac{a+b}{a} = \frac{a}{b} \qquad \Phi = \frac{a}{b}$

$\frac{a+b}{a} = \frac{a}{b} \qquad \frac{b}{a} = \frac{1}{\Phi}$

↗

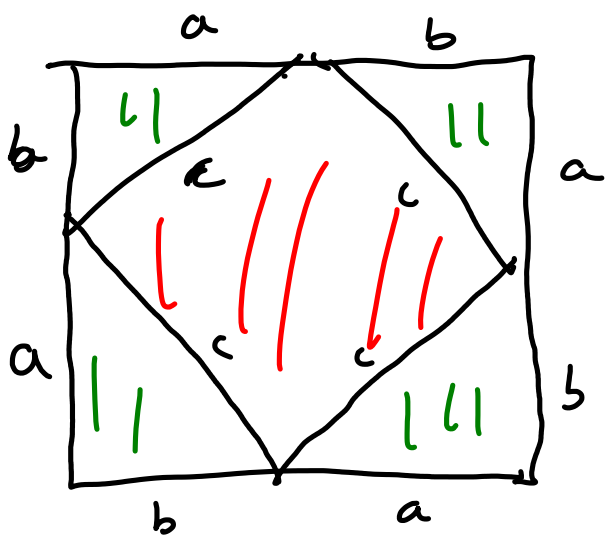
↘ $\frac{a}{a} + \frac{b}{a} = \frac{a}{b} = 1 + \frac{b}{a}$

$\Phi = 1 + \frac{1}{\Phi}$

$\Phi^2 - \Phi - 1 = 0$

$\Phi = \frac{1 + \sqrt{5}}{2} = \frac{a}{b}$

3 a)

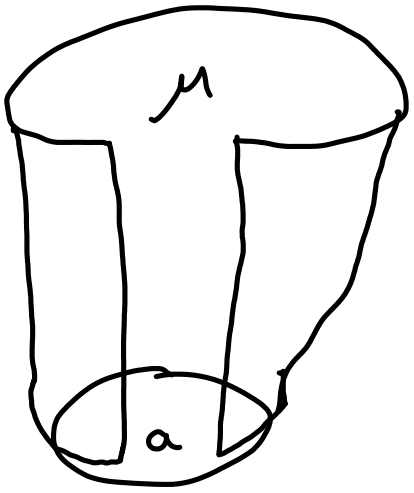


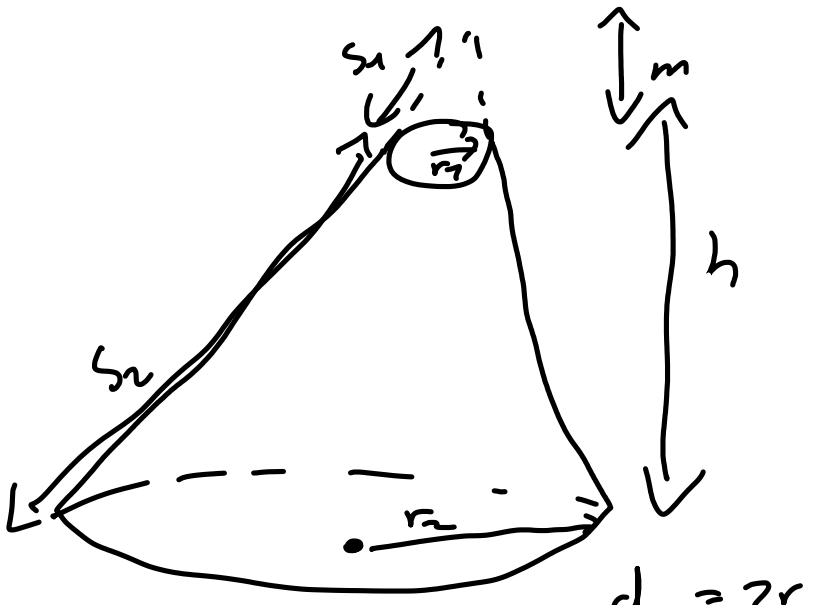
$(a+b)^2 = 4 \cdot \frac{a \cdot b}{2} + c^2$

$a^2 + 2ab + b^2 = 2ab + c^2$

$a^2 + b^2 = c^2$

b)





$d_2 = 2r_2$

$d_1 = 2r_1$

$$V = \frac{h\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)$$

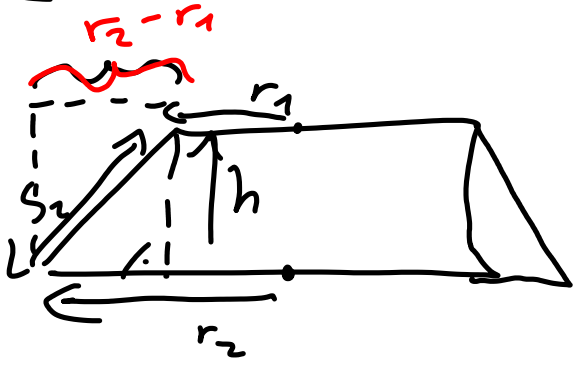
$\approx 10{,}5\,L$

$1\,cm^3 = (10^{-2})^3\,m^3 = 10^{-6}\,m^3$

$\Rightarrow 1\,m^3 = 1000\,L$

$1\,cm^3 = 10^{-3}\,L$

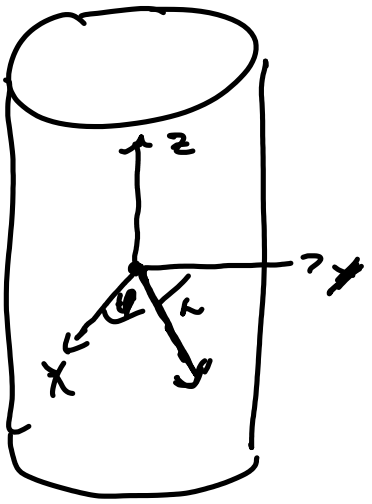
Mantelfläche:



$$s = \sqrt{(r_2 - r_1)^2 + h^2}$$

$M = (r_2 + r_1)\,\pi \cdot s = 0{,}1997\,m^2$

c) (x, y, z) in (r, φ, z)

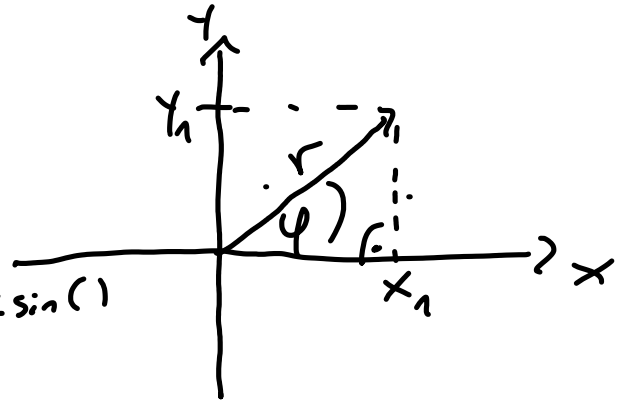


$(1, 0, 0)$: $x = 1$
$y = 0$
$z = 0$

$\Rightarrow$ $z = 0$

$r = \sqrt{x^2 + y^2}$
$= \sqrt{1+0} = 1$

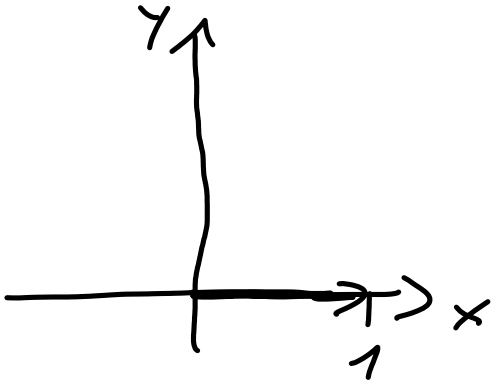
$\varphi =$



$\sin(\varphi) = \frac{y_1}{r}$ | arcsin()

$\cos(\varphi) = \frac{x_1}{r}$ | arccos()

$\varphi = \arcsin\left(\frac{y_1}{r}\right)$
$= \arcsin\left(\frac{x_1}{r}\right) = 0$

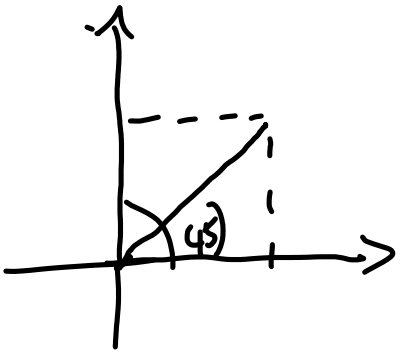


Zylinderkoord.: $(1, 0, 0)$

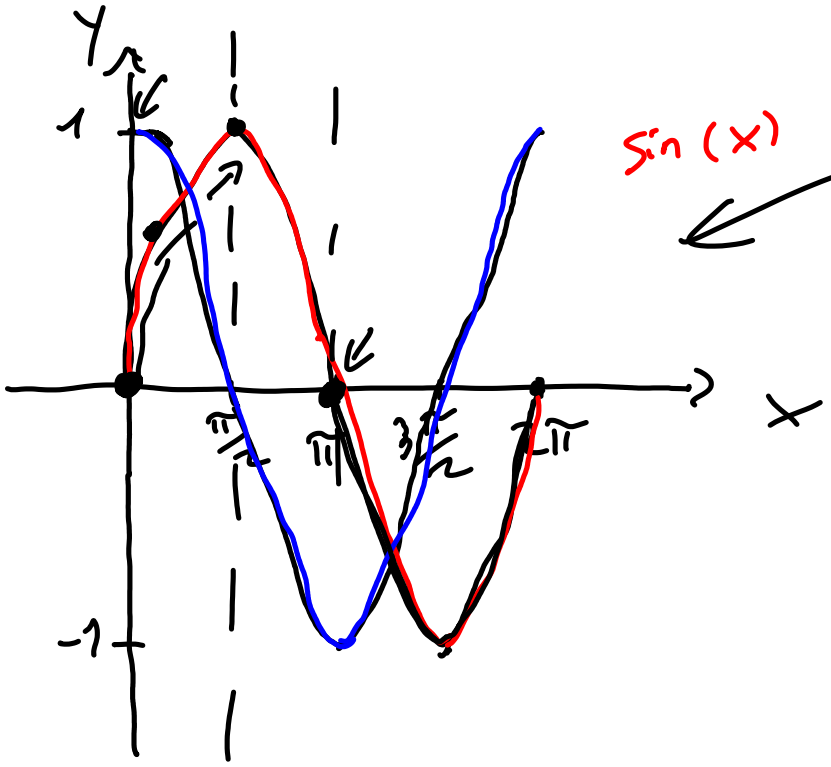
b) $(1, 1, 0)$ $z = 0$

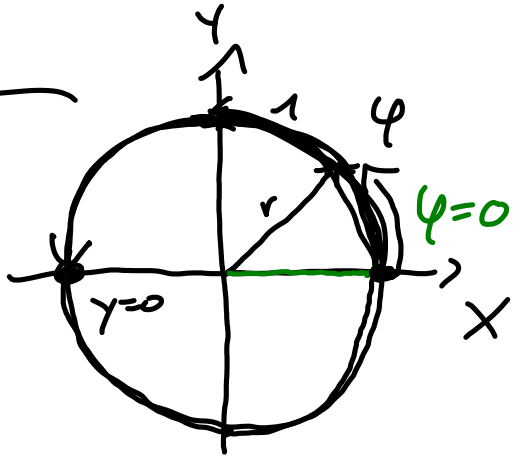
$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\varphi = \frac{\pi}{4}$



$f(x) = x$





$\sin(\varphi) = \frac{x}{r}$

$\sin(\varphi) = y \overset{\varphi=0}{=} 0$

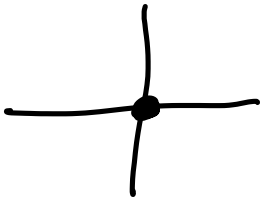
$\sin 90° = \frac{\pi}{2}$

Kood: (2, 3, 4)

$r = \sqrt{(2)^2 + (3)^2}$ $\quad z = 4$

$\varphi = \arcsin\left(\frac{3}{r}\right)$

f) (0, 0, 0) $\quad z = 0$



$r = \sqrt{0^2 + 0^2} = 0$

$\varphi = \arcsin\left(\frac{x}{r}\right)$

Zylinder (0, 0, 0)

4) Newton

2 Newton Gesetz $F = m \cdot a \Rightarrow \sum F = 1\,kg \frac{m}{s^2} = 1\,N$

(m: Masse kg; a: Beschleunigung $\frac{m}{s^2}$)

b) Joule $E_{kin} = \frac{1}{2} m v^2 \rightarrow [E] = 1\,kg \cdot \frac{m^2}{s^2} = 1\,J$

Masse kg, Geschwind $\frac{m}{s} \Rightarrow [v^2] = \frac{m^2}{s^2}$

c) $\cdot q$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3} \qquad [q] = C \qquad [r] = m$$

$[\varepsilon_0] = 1 \frac{As}{Vm}$

$$[\vec{E}] = 1 \frac{C}{\frac{As}{Vm}} \cdot \frac{m}{m^2} = 1\,C \frac{Vm}{As m^2} = 1 \frac{V}{m}$$

$= 1\,V/m$

Strom $[I] = 1A = 1 \frac{C}{s}$

d)

a) $2,35 \cdot 10^{10}\,dm^2$	a) $9,837 \cdot 10^{-2}\,cm^3$
b) $28,7\,dm^2$	b) $5,32 \cdot 10^4\,cm^3$
c) $34,27\,dm^2$	c) $3,45 \cdot 10^4\,cm^3$
d) $8,342 \cdot 10^3\,dm^2$	d) $200\,cm^3$
e) $3,648 \cdot 10^7\,dm^2$	e) $4,37 \cdot 10^5\,cm^3$
f) $2 \cdot 10^{-8}\,dm^2$	f) $4,5 \cdot 10^7\,cm^3$

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