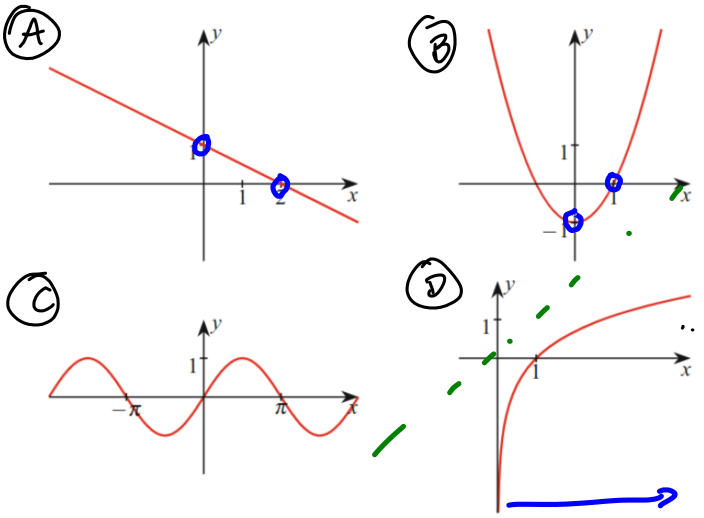
Vorrechnen Blatt 3 06.10.2022

Aufgabe 1: Einfache Funktionen

(i)



A $f(x) = ax + b \rightarrow b = 1$

$f(x) = -\frac{1}{2}x + 1$

$f: \mathbb{R} \rightarrow \mathbb{R}$, bijektiv

B $f(x) = x^2 - 1$

$D = \mathbb{R}$, $W = [-1, \infty[$, $[-1, \infty)$

bijektiv auf $[0, \infty) \rightarrow [-1, \infty)$

C $\sin(x)$ bijektiv auf $[-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$

$D = \mathbb{R}$, $W = [-1, 1]$

D $\ln(x)$ bijektiv auf $\mathbb{R}^+ \rightarrow \mathbb{R}$

(ii) $f(x) = x^2 + 2x - 15$

a) Definitionsbereich : $\mathbb{R}$

Wertebereich: $[-16, +\infty)$

$$x^2 + 2x = (x+1)^2 - 1 \rightarrow f(x) = (x+1)^2 - 1 - 15$$

$$= \underbrace{(x+1)^2}_{\geq 0} - 16 \geq -16$$

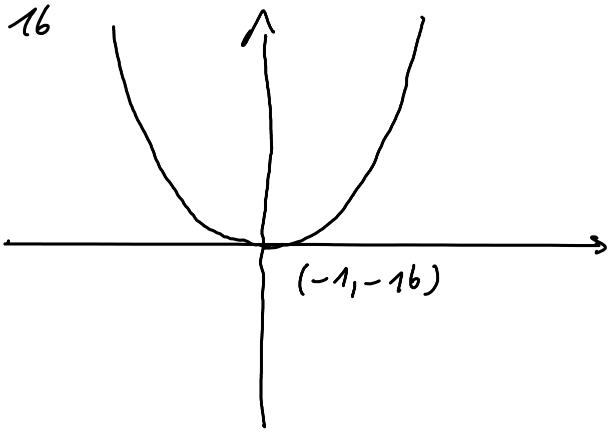
Nullstellen: $x^2 + 2x - 15 = 0 \Leftrightarrow (x+1)^2 - 16 = 0$

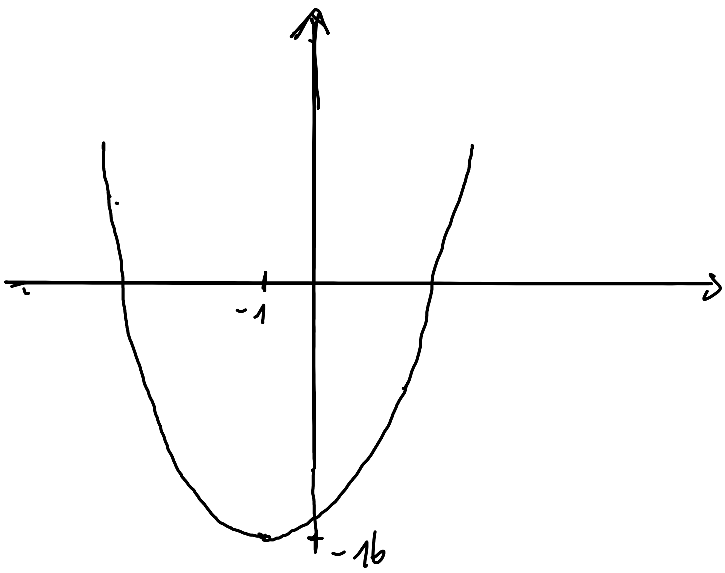
$$\Rightarrow (x+1)^2 = 16 \Rightarrow x+1 = \pm\sqrt{16} = \pm 4$$

$$\Rightarrow x = -1 \pm 4 = -5 \vee 3$$

b) $f(x) = (x-3)(x+5)$

c) $f(x) = (x+1)^2 - 16$



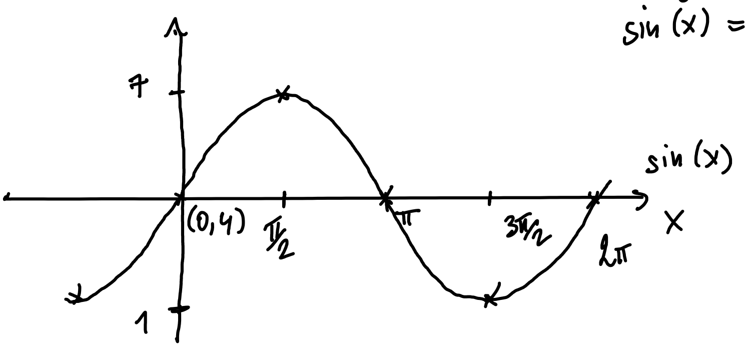


d) monoton fallend auf $x \leq -1$

monoton steigend auf $x \geq -1$

(iii)

a) $f(x) = 3 \cdot \sin(x) + 4$



allgemein:

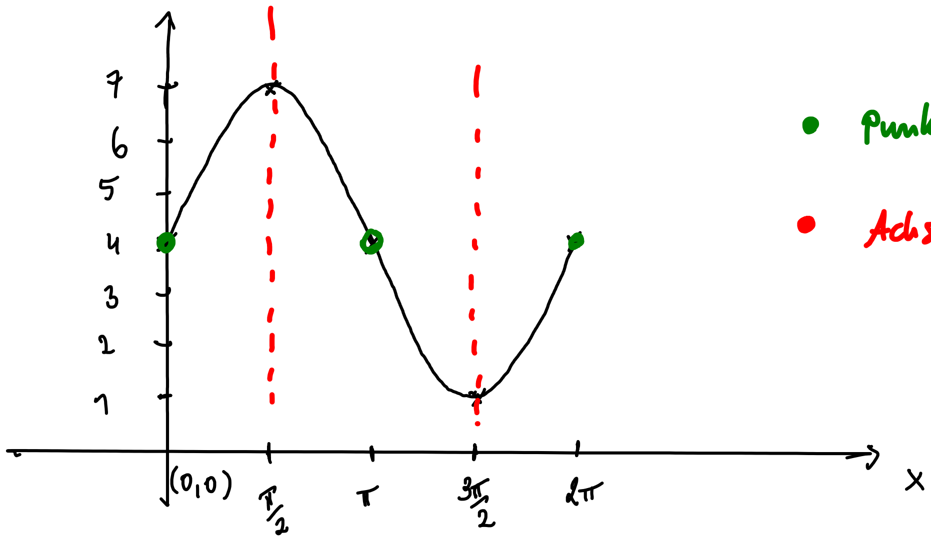
$\sin(x) = 0 \Rightarrow x = z \cdot \pi \,, \quad z \in \mathbb{Z}$

Maxima

$\sin(x) = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi \cdot z$

Minima

$\sin(x) = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi \cdot z$

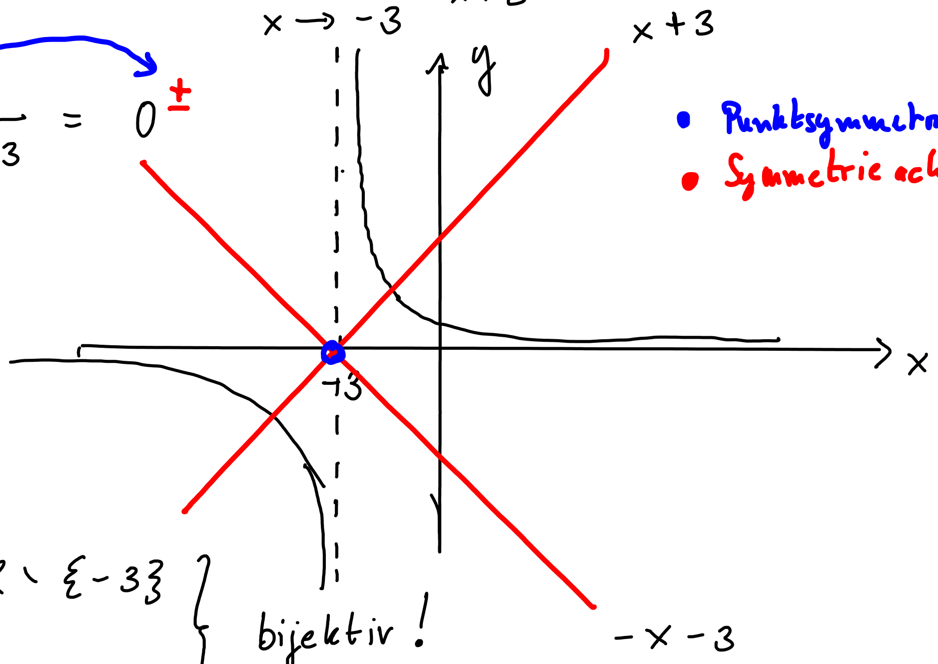


- Punktsymmetrie
- Achsensymmetrie

b) $f(x) = \frac{1}{x+3} \qquad D = \mathbb{R} \setminus \{-3\}$

$\lim_{x \to -3^{+}} \frac{1}{x+3} = +\infty \qquad \lim_{x \to -3^{-}} \frac{1}{x+3} = -\infty$

$\lim_{x \to \pm\infty} \frac{1}{x+3} = 0^{\pm}$



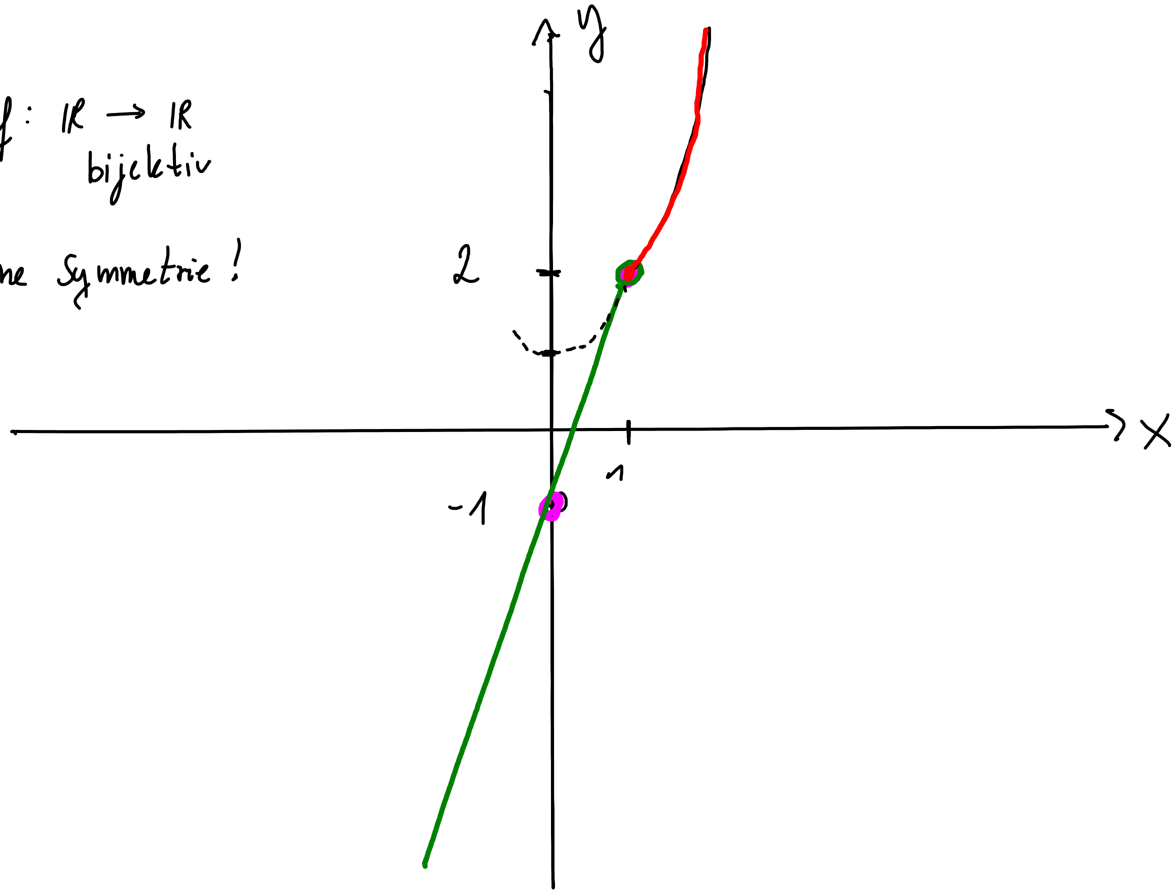
- Punktsymmetrie
- Symmetrieachsen

Definitionsmenge : $\mathbb{R} \setminus \{-3\}$

Wertemenge : $\mathbb{R} \setminus \{0\}$

} bijektiv!

c) $f(x) = \begin{cases} 3x-1, & x < 1 \\ x^2+1, & x \geq 1 \end{cases}$



$f: \mathbb{R} \to \mathbb{R}$
bijektiv

keine Symmetrie!

(iv)

a) $f(x) = x \cdot \sin(x)$

Punktsymmetrie : $f(-x) = -f(x)$

Achsensymmetrie : $f(-x) = f(x)$

$$f(-x) = -x \cdot \underbrace{\sin(-x)}_{= -\sin(x)} = x \cdot \sin(x) = f(x) \Rightarrow \text{AS}$$

b) $f(x) = \dfrac{e^x + e^{-x}}{2}$

$$f(-x) = \frac{e^{-x} + e^x}{2} = f(x) \Rightarrow \text{AS}$$

c) $f(x) = \dfrac{(x^5 + 4x^3 + 2x)\ \sin^2(x)}{|x| \cdot \cos(x)}$

$$f(-x) = \underbrace{((-x)^5 + 4(-x)^3 - 2x)}_{-(x^5+4x^3+2x)} \underbrace{(-\sin(x))^2}_{(\sin(x))^2} \cdot \frac{1}{\underbrace{|-x|}_{|x|} \cdot \underbrace{\cos(-x)}_{\cos(x)}}$$

$$\Rightarrow \quad -\frac{(x^5+4x^3+2x)\ \sin^2(x)}{|x|\cdot \cos(x)} = -f(x)$$

$$\Rightarrow \quad PS$$

d) $f(x) = (x+8)^3 - (x-8)^3$

$f(-x) = (-x+8)^3 - (-x-8)^3 = -(x-8)^3 + (x+8)^3 = f(x)$

$\Rightarrow$ AS

e) $f(x) = (x+8)^2 - (x-8)^2 = (-x+8)^2 - (-x-8)^2$

$= (-(x-8))^2 - (-(x+8))^2$

$= (x-8)^2 - (x+8)^2 = -[(x+8)^2 - (x-8)^2]$

$= -f(x)$

$\Rightarrow$ PS

f) $f(x) = \ln(\sqrt{x^2+1} + x)$

$f(x) + f(-x) = \ln(\sqrt{x^2+1} + x) + \ln(\sqrt{x^2+1} - x)$

$= \ln((\sqrt{x^2+1}+x)\cdot(\sqrt{x^2+1}-x))$

$= \ln(x^2+1 - x\cancel{\sqrt{x^2+1}} + x\cancel{\sqrt{x^2+1}} - x^2)$

$= \ln(1) = 0$

$\Rightarrow f(x) + f(-x) = 0 \Rightarrow f(-x) = -f(x) \Rightarrow$ PS

$(x+a)\cdot(x+b) = x^2 + x\cdot b + a\cdot x + ab$

Aufgabe 2: Verkettung von Funktionen

$f(x) = 4x^2 - 4x + 4, \quad g(x) = x - 2$

$\boxed{x^2 + ax = (x + a/2)^2 - \frac{a^2}{4}}$

$f(x) = 4x^2 - 4x + 4 = 4\cdot(x^2 - x + 1)$

$x^2 + 2\cdot\left(\frac{a}{2}\right)x + \frac{a^2}{4}$ — a

NR: $x^2 - x + 1 = (x - \frac{1}{2})^2 - \frac{1}{4} + 1$

$= (x - \frac{1}{2})^2 + \frac{3}{4}$

$$\to f(x) = 4\left(x-\tfrac{1}{2}\right)^2 + 3 \;:\; \mathbb{R} \to [3,\infty)$$

$(x+a)(x+a)$ $2ax$

$x^2 + \overbrace{x \cdot a + a \cdot x}$

$+a^2$

$$g(x) = x-2 \;:\; \mathbb{R} \to \mathbb{R}$$

$$(f\circ g)(x) = f(g(x)) = 4(x-2)^2 - 4(x-2) + 4$$

$$= 4(x^2-4x+4) - 4x + 8 + 4$$

$x^2 + 2ax + a^2$

$$= 4x^2 - 16x + 16 - 4x + 8 + 4$$

$a = \frac{c}{2}$

$$= 4x^2 - 20x + 28$$

$x^2 + cx + \left(\frac{c}{2}\right)^2$

$$= 4\cdot(\underline{x^2 - 5x} + 7)$$

$$x^2 - 5x = \left(x-\tfrac{5}{2}\right)^2 - \frac{25}{4}$$

$$= 4\cdot\left(\left(x-5/2\right)^2 \underbrace{- \frac{25}{4} + 7}_{\frac{3}{4}}\right)$$

$$7 = \frac{28}{4}$$

$$= 4\cdot(x-5/2)^2 + 3$$

$$(f\circ g)(x) \;:\; \mathbb{R} \to [3,\infty)$$

$$(g\circ f)(x) = 4x^2 - 4x + 4 - 2$$

$$= 4\cdot\left(x-\tfrac{1}{2}\right)^2 \underbrace{+3-2}_{1} = 4\cdot\left(x-\tfrac{1}{2}\right)^2 + 1$$

$$(g\circ f)(x) \;:\; \mathbb{R} \to [1,\infty)$$